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Phase space simulations of the Novosibirsk/SELENE FEL

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ABSTRACT

SELENE (**S**pac**E** **L**aser **ENE**ergy) is a proposal to use a free electron laser (FEL) as a ground-based power source for several space applications. An advanced FEL utilizing a multisection optical klystron coupled to a single pass radiator has been suggested. Phase space simulations show electron bunching at low optical power in the klystron oscillator, leading to high gain in the single pass radiator.

1. INTRODUCTION

In the SELENE proposal, electrical power is converted to optical power using an FEL. The optical power is then directed to an adaptive optics telescope which prepares the wavefront and redirects the beam into space. The beam travels either directly to its end user or to a beacon located approximately 25 km above the telescope which would then redirect it to the user. The ability to make additional power available in space without the use of conventional heavy-lift vehicles has great economic potential. [1]

An FEL for the SELENE project has been proposed by Dr. Vinokurov of the Budker Institute of Nuclear Physics, Novosibirsk, Russia. This FEL is composed of a four section optical klystron coupled to a single pass radiator as shown in figure (1). The undulators used within the klystron and the radiator are identical. The klystron will be supplied an electron beam on the order of 100 MeV from a four pass race-track microtron-recuperator (RTMR). The optical wavelength, $\lambda = 0.84\ \mu m$, was chosen for low atmospheric absorption and high solar cell efficiency. The initial project goal is to achieve 200 kW average power from the FEL with an ultimate goal of 10 MW. The prototype for this FEL is currently being designed and constructed in Novosibirsk. [2]

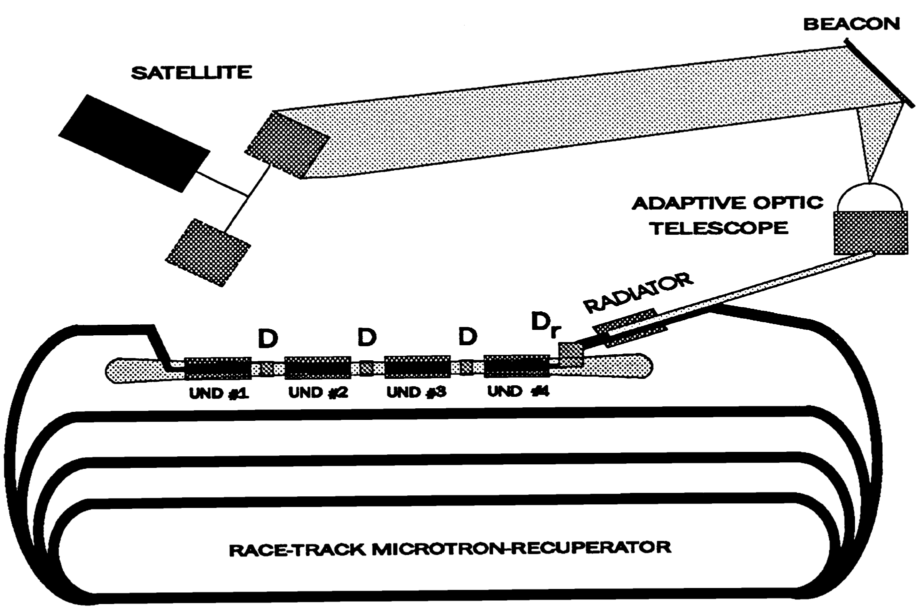

Figure 1. SELENE concept incorporating Novosibirsk proposal for a race-track microtron-recuperator, four section FEL klystron and single pass radiator.

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2. OPTICAL KLYSTRON

The FEL klystron operates much the same as a basic FEL. The electron trajectories are governed by the undulator's static magnetic field, and their evolution is determined from the relativistic Lorentz force equations,

$$\frac{d\gamma(\tau)}{d\tau} = \frac{eKLE(\tau)}{\gamma mc^2} cos[\zeta(\tau) + \phi(\tau)] \quad , \tag{1}$$

$$\frac{d\zeta(\tau)}{d\tau} = \nu(\tau) \approx L\left[k_o - \frac{k(1+K^2)}{2\gamma^2}\right] \quad , \tag{2}$$

where the dimensionless time is $\tau = tc/L$. The total undulator length is $L = N\lambda_o$, with N being the total number of undulator periods within the klystron. The electron position along the undulator axis is $z(t)$, and its phase with respect to the FEL klystron optical field is $\zeta(\tau) = (k + k_o)z(t) - \omega t$, where the optical frequency is $\omega = 2\pi c/\lambda$. The relativistic Lorentz factor is γ, e is the electron charge, $K = e\bar{B}\lambda_o/2\pi mc^2$ is the undulator parameter, $\bar{B}$ is the undulator rms magnetic field strength, λ_o is the undulator period, m is the electron mass and c is the speed of light. The slowly evolving optical field strength is $E(\tau)$ and $\phi(\tau)$ is the slowly evolving optical phase. The electron phase velocity is $\nu(\tau)$, and the optical and undulator wavenumbers are $k = 2\pi/\lambda$ and $k_o = 2\pi/\lambda_o$. Equation (1) describes the evolution of the electron energy, and equation (2) is the evolution of the dimensionless phase velocity when $\gamma \gg 1$. The FEL operates near the resonance condition $\nu \approx 0$ which is met when $\lambda \approx \lambda_o(1 + K^2)/2\gamma^2$. Equations (1) and (2) taken together describe electron motion in the presence of the evolving field $[E(\tau), \phi(\tau)]$.

When $N \gg 1$, the electron energy is nearly constant and (1) and (2) result in the pendulum equation,

$$\frac{d^2\zeta(\tau)}{d^2\tau} = \frac{d\nu(\tau)}{d\tau} = |a(\tau)| cos[\zeta(\tau) + \phi(\tau)] \quad , \tag{3}$$

where $|a(\tau)| = 4\pi NeKLE/\gamma^2 mc^2$ is the amplitude of the dimensionless self-consistent optical field. Equation (3) describes the electrons' phase space motion inside the undulators of the FEL klystron.

The dispersive sections between the undulators cause the electrons to evolve differently than when they are in the undulators. The large transverse motions that occur within the dispersive sections result in the electrons moving far away from resonance and, therefore, they do not interact with the optical field during that time. The only changes in the electron trajectories come from the phase velocity differences after exiting the previous undulator sections. These differences are translated into changes in the electron phase, ζ, so that the net effect of the dispersive section is

$$\Delta\zeta = \nu D, \quad \Delta\nu = 0 \quad , \tag{4}$$

where D is the dimensionless drift time. The strength of the dispersive magnets is measured by $D = \int B(z')dz'$ within the dispersive magnet. The phase velocity, ν, is unaffected by the dispersive section because there is no interaction with the optical field.

The self-consistent wave equation governing the evolution of the complex optical field $a = |a|e^{i\phi}$ is

$$\frac{da(\tau)}{d\tau} = -j\langle e^{-i\zeta(\tau)}\rangle \quad , \tag{5}$$

where j is the dimensionless current density

$$j = \frac{8N(e\pi KL)^2\rho}{\gamma^3 mc^2} \quad , \tag{6}$$

where $\rho = 3\times10^9 I / ec\pi r_b^2$ is the electron density in the beam, the electron beam current I is in Amperes, and πr_b^2 is the electron beam area. The quantity inside the brackets, <...>, is the average over the electrons. Initially, the electrons are uniformly distributed, populating all values of ζ_o from 0 to 2π so the average, <...>, is zero. As bunching occurs due to interaction, the average begins to grow, forcing the field $a(\tau)$ to grow.

When the value of $D > 0$, large gain can be achieved with lower values of j compared to the conventional FEL. For a linear undulator, the optical field goes as $a(\tau) \rightarrow a(\tau)[J_o(\xi) - J_1(\xi)]$ and the dimensionless current density goes as $j \rightarrow j\,[J_o(\xi) - J_1(\xi)]^2$, where $\xi = K^2/2(1 + K^2)$. An additional factor affecting the dimensionless current density is the "filling factor", $F = \pi r_b^2 / \pi\omega_o^2$ which describes the cross-section overlap between the electron beam and the optical mode reducing $j \rightarrow jF$. [3]

3. NOVOSIBIRSK FEL PROTOTYPE

The Novosibirsk 51 MeV RTMR [4] will provide a beam of relativistic electrons to the FEL klystron. This electron beam consists of micropulses of 20 - 100 ps in length with a repetition frequency of 2 - 45 MHz and a peak current of 20 - 100 A. The FEL klystron consists of three identical undulators of length $N\lambda_o$ where the undulator wavelength is $\lambda_o = 9$ cm and the number of undulator periods is $N = 40$. It is proposed that each undulator is separated from the next by dispersive magnets equivalent to $D = 0.5$. The optical resonator mirrors located at either end of the undulator are 79 m apart. The single pass radiator is an identical undulator separated from the klystron by a dispersive magnet equivalent to $D = 0.25$ which imparts a 4mrad bend on the electron beam. This bend removes the beam from the klystron and injects it into the radiator. The radiator extracts energy from the bunched electron beam beginning with spontaneous emission. The efficiency is limited to about $\eta \approx 2$ % so that the electron beam may be reintroduced into the RTMR for the recuperator phase.

The prototype has $N = 160$ undulator periods for the system (radiator and klystron), or $N = 120$ for the optical klystron alone. The undulator parameter $K = 1.4$ corresponds to an optical wavelength of $\lambda = 13$ µm with Lorentz factor $\gamma = 100$. This value of K results in $\xi = 0.3$. The total length of the undulators is $L = 1440$ cm for the system, or $L = 1080$ cm for the optical klystron. The electron beam current is $I = 100$ A, and the beam area is $\pi r_b^2 = 0.03$ cm^2. The average filling factor is $\bar{F} = 0.027$ for the system and $\bar{F} = 0.035$ for the optical klystron. These values give a dimensionless current density of $j = 180$ when simulating the entire system, and $j = 100$ when simulating just the klystron oscillator.

The dimensionless klystron strength, or "drift time" D, can be varied to maximize the power out of the radiator while minimizing the power within the klystron. Initially, $D = 0.5$ for each dispersive section within the klystron and $D_r = 0.25$ for the dispersive section separating the klystron and the radiator. When simulating the klystron alone, $D = 0.67$ and $D_r = 0.33$ are used to account for the shorter total undulator length, since the dimensionless parameters are scaled to this length. D_r is still used with the klystron to provide an indication of the electron bunching that has taken place just prior to entering the radiator.

4. OPTICAL FIELD STRENGTH LIMITATION

The FEL klystron is designed to create electron bunching in weak optical fields. This is important because strong optical fields in the klystron oscillator might cause mirror damage. Here we will calculate the maximum allowed optical field strength in the klystron oscillator.

A wide range of mirror material can be used, each having its own maximum power density limit. These limits vary from 10–100 kW/cm^2. A maximum power density of $P_M = 5$ kW/cm^2 is assumed in order to assure a safety margin independent of the mirror type.

The dimensionless optical field strength at the mirrors is determined through the relation,

$$|a| = \frac{4\pi NeKLE}{\gamma^2 mc^2} = \frac{4\pi NeKL}{\gamma^2 mc^2}\left[\frac{8\pi P_M}{c}\right]^{\frac{1}{2}} \quad , \tag{7}$$

where $P_M = E^2 c / 8\pi$ is the optical power density. The beam spot size at the mirror w is related to the beam spot size at the waist w_o by

$$w^2 = w_o^2\left[1 + \frac{S^2}{4z_o^2}\right] \quad , \tag{8}$$

where the mirror separation is $S = 79$ m, and the resonator Rayleigh length is $z_o = \pi w_o^2/\lambda$. Conservation of energy requires that $w_o^2 a_o^2 = w^2 a^2$. The maximum optical field strength within the FEL klystron is then

$$a_o = \left[1 + \frac{S^2}{4z_o^2}\right]^{\frac{1}{2}} a = \left[1 + \frac{S^2}{4z_o^2}\right]^{\frac{1}{2}} \frac{4\pi NeKL}{\gamma^2 mc^2}\left[\frac{8\pi P_M}{c}\right]^{\frac{1}{2}} \quad , \tag{9}$$

Using an optical beam waist of $w_o = 2r_b$ and an optical wavelength of $\lambda = 13$ µm for the prototype FEL, the Rayleigh length is $z_o = 4$ m. Equation (9) then gives a maximum dimensionless optical field strength within the klystron of $a_o \approx 8$. This is well into the strong field regime, $a_o > \pi$,for both a normal FEL and an FEL klystron. In an FEL klystron, good electron bunching can be achieved at much lower values of a_o if the electron beam quality is sufficient.

5. ELECTRON BEAM QUALITY

The electron beam quality from the RTMR can be described by the emittance, $\varepsilon = r_b\bar{\theta}$ where r_b is the rms initial electron radial position and $\bar{\theta}$ is the rms initial angular spread of electrons away from the z axis. Either rms value can be altered by external focusing, but the product ε is fixed. The angular and position spreads are matched to prevent excess focusing or expanding of the beam along the z axis by requiring $Kk_o r_b \approx \gamma\bar{\theta}$. This results in a maximum emittance of $\varepsilon \approx \gamma\lambda / 2\pi NK \approx 0.2\pi$ mm-mrad for this system before the gain is severely reduced. [5] The design emittance is $\varepsilon \approx 0.2\pi$ mm-mrad. [4]

Beam quality is also characterized by the spread in energy of the electrons. The initial dimensionless phase velocity of the electron, ν_o, depends on the square of the electron energy so that a spread in electron energy will result in a spread in initial electron phase velocities. The design energy spread is $\Delta\gamma/\gamma = 0.45$ % leading to a spread in initial electron phase velocities of $\Delta\nu = 4\pi N\Delta\gamma/\gamma \approx 9$. [4]

6. SYSTEM SIMULATIONS

One-dimensional computer simulations of the FEL klystron and radiator system were conducted to determine the sensitivity of the design to the electron energy spread, $\Delta\gamma/\gamma$, and to the initial angular spread of electrons, $\bar{\theta}$, which forces a spread in initial electron position due to the fixed value of emittance.

The spread in electron energy is represented by a Gaussian distribution in phase velocity about ν_o having a standard deviation of $\sigma_G = 4\pi N\Delta\gamma/\gamma$. A Gaussian distribution in electron injection angles, assuming a matched beam, is equivalent to an exponential distribution of phase velocities with characteristic width

$\sigma_\theta = 4\pi N\gamma^2\bar{\theta}^2/(1+K^2)$. Using various values of σ_G and σ_θ, simulations provide an indication of the system's sensitivity to electron beam energy spread and emittance.

Figure 2 shows the simulation results for a single pass through the oscillator and the radiator with a mono-energetic beam and perfect beam injection ($\sigma_G = \sigma_\theta = 0$). The initial optical field amplitude, $a_o = 0.1$, is in the weak-field regime. The dimensionless phase velocity, $\nu_o = 0$, results in maximum gain for the given parameters. The dimensionless current density in the radiator is $j_r = 3500$. This value was determined by assuming a filling factor for the radiator of about $F \approx 1/3$ based on three-dimensional simulations including the effects of optical guiding.[5] The left plot shows the final position of electrons in phase-space. The upper and lower curves indicate the separatrix which delineates the regions of open and closed phase-space paths. This simulation utilizes 40 sample electrons and plots their final positions. The top right plot shows the optical power evolution. The vertical axis is the natural log of the dimensionless power which is defined as $p = |a|^2$. The lower plot show the evolution of the optical phase ϕ.

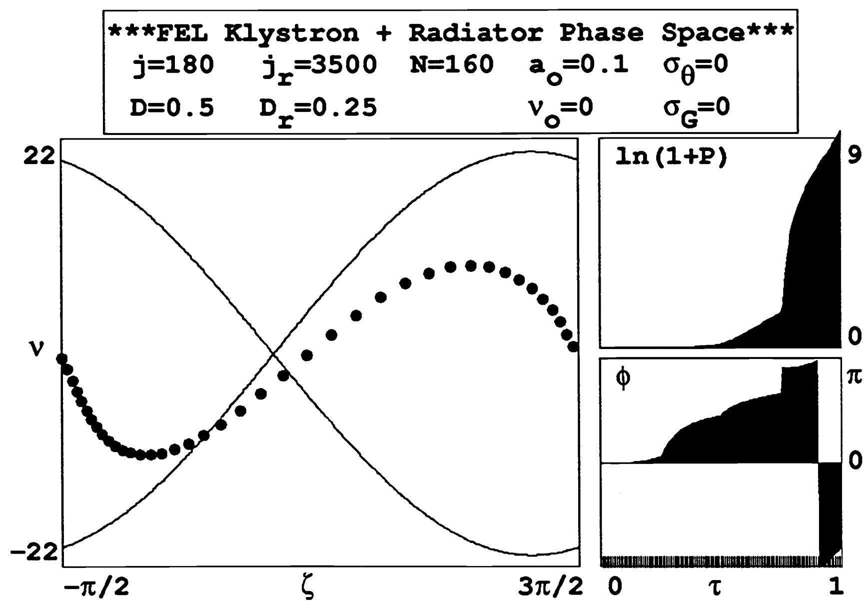

Figure 2. Phase-space simulation for prototype system with a perfect electron beam.

There is some increase in optical power as the electrons pass through the FEL klystron from $\tau = 0$ to $\tau = 0.75$. When the electrons enter the radiator at time $\tau = 0.75$, the optical field is reset to zero, but then it rapidly grows from spontaneous emission and amplification by the bunched electron beam, as seen in the power evolution from $\tau = 0.75 \rightarrow 1.0$. The final dimensionless power is about $p = 11{,}600$ which is equivalent to a peak power density of 77 MW/cm^2. A duty factor of 0.0009 yields an average power density output of 69 kW/cm^2 for a perfect electron beam.

Values of σ_G and σ_θ that would each, independently, cause the final power out of the radiator to drop by about half were sought to provide a measure of sensitivity of the system. Figure 3 shows the simulation results using an electron beam energy spread $\Delta\gamma/\gamma = 0.04\%$, or $\sigma_G = 0.8$. The initial phase velocity has been adjusted to maintain maximum gain. A total of 100,000 sample electrons are used to limit the sensitivity of the simulation to shot noise. The final phase-space positions of 500 sample electrons are plotted giving a good indication of their spread. The net energy transfer from the electrons to the optical wave has been reduced. The final power has dropped to about $p \approx 5000$, which gives an average power density of 30 kW/cm^2, about half of the value obtained with a perfect electron beam. This indicates that the prototype FEL klystron will be very sensitive to electron beam energy spread. The SELENE design with an additional undulator section in the FEL klystron would require an even smaller energy spread.

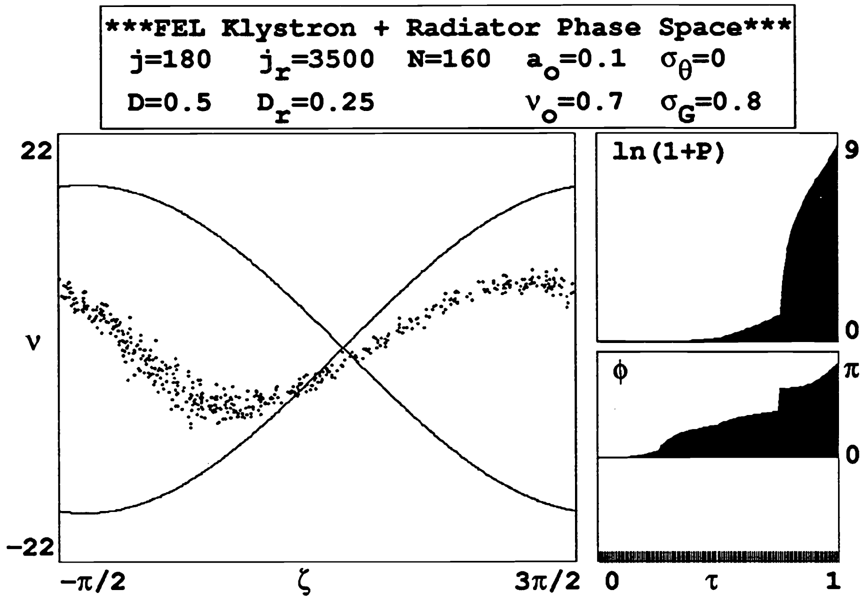

Figure 3. Phase-space simulation for prototype system with electron beam energy spread sufficient to reduce output by half.

Figure 4 shows the simulation results for a mono-energetic electron beam with a spread of injection angles given by $\sigma_\theta = 0.7$, which corresponds to an rms injection angle of $\overline{\theta} = 0.32$ mrad. The final power $p \approx 5600$ is again reduced by half from the perfect beam case. The fixed value of emittance, $\varepsilon = 0.2\pi$ mm-mrad leads to an rms initial position spread of $r_b = 2.0$ mm.

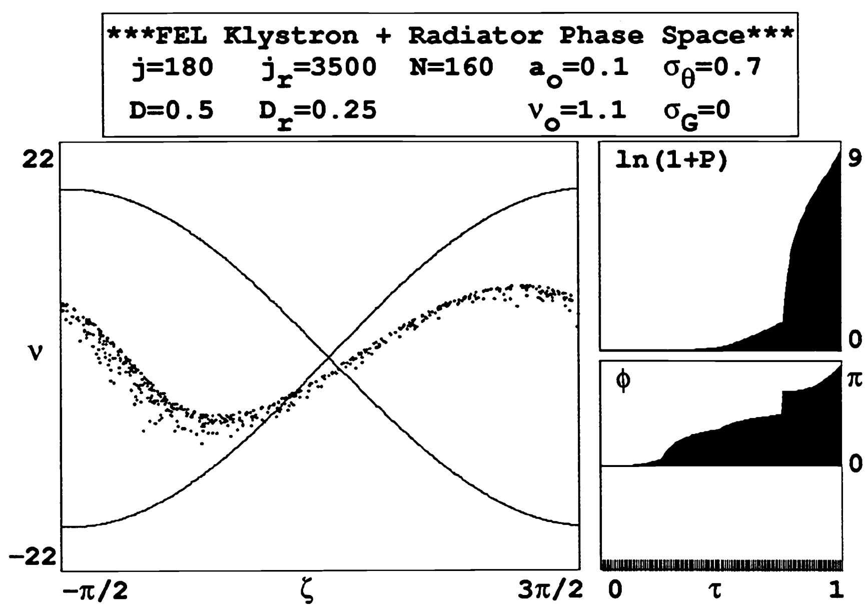

Figure 4. Phase-space simulation of prototype system with electron beam injection angle sufficient to reduce output by half.

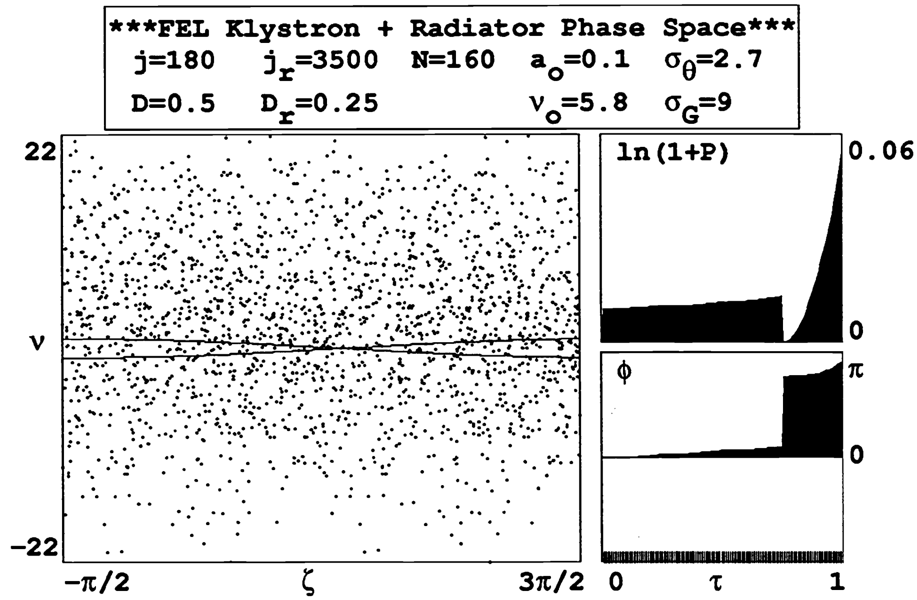

Figure 5. Phase-space simulation for prototype system with initial parameters for electron beam energy spread and injection angle.

The FEL klystron and radiator prototype uses estimated parameters. Figure 5 is a phase-space plot of the system using these values. The value of $\sigma_\theta = 2.6$ was calculated assuming an electron beam radius of $r_b = 1$ mm and the emittance fixed at $\varepsilon = 0.2\pi$ mm-mrad. The dimensionless phase velocity ν_o has been adjusted to optimize gain as would naturally occur in the FEL klystron. The final electron positions in phase-space are almost entirely random with no apparent bunching. This is strongly influenced by the system sensitivity to the large electron beam energy spread. The final power has decreased dramatically to $p \approx 0.06$ which gives an average power density of 0.36 $\mathrm{W/cm^2}$.

7. SUMMARY

The prototype SELENE proposal uses an FEL klystron for electron bunching. This allows the optical field strengths within the klystron to be kept low. However, the multisection klystron is very sensitive to the electron beam energy spread. Adding more undulators would increase the sensitivity of the system. The power density limitations within the FEL klystron allow for strong optical fields if desired. This may allow the use of a simple undulator to provide the electron bunching reducing the sensitivity to energy spread. More simulations will be used to determine the best possible combination of parameters. Some of the parameters that can be changed are: the strengths of each dispersive magnet, the undulator lengths, the number of undulators within the FEL klystron, and the undulator field strength and the length of the radiator undulator. The large number of variables could result in more than one "best design".

8. REFERENCES

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